What is the Upper Limit for Power Production From a Tidal Channel?

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Sea change for tidal power

New underwater turbines could be cheap and eco-friendly.

24 March 2004

MARK PEPLOW

A British company has invented a simple tidal power system

www.sciencemag.org SCIENCE VOL 299 17 JANUARY 2003

Renewable Energy

Norway Goes With the Flow To Light Up Its Nights

Three European teams are racing to be the first in the world to harness a new source of power: underwater coastal currents driven by the tides

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Published online: 13 August 2004; | doi:10.1038/news040809-17

Tidal flow to power New York City

Helen Pearson

Green energy firm plans turbines in the East River.

Uldolmok

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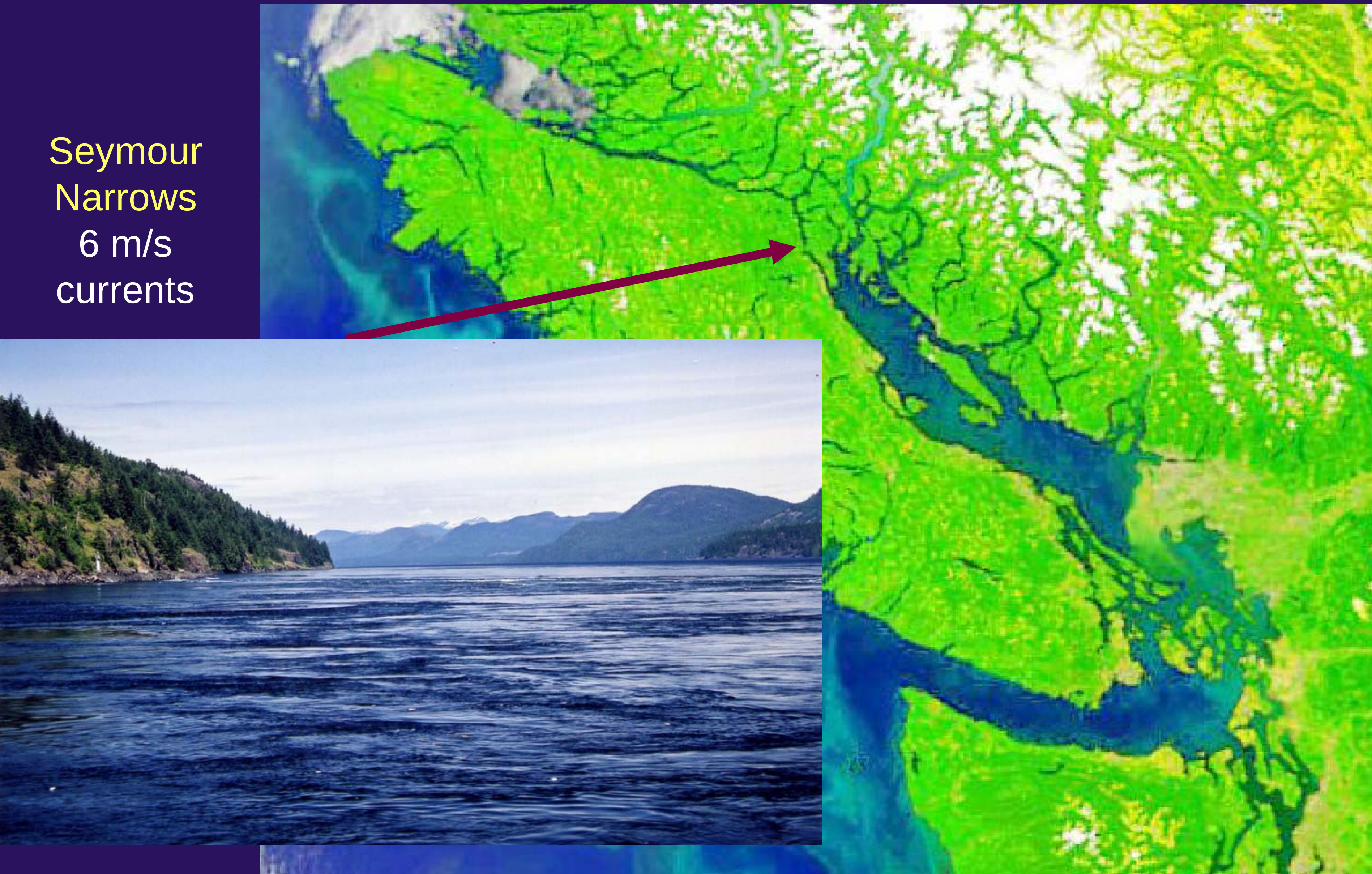
What is the power potential of local currents?

Seymour Narrows
6 m/s currents



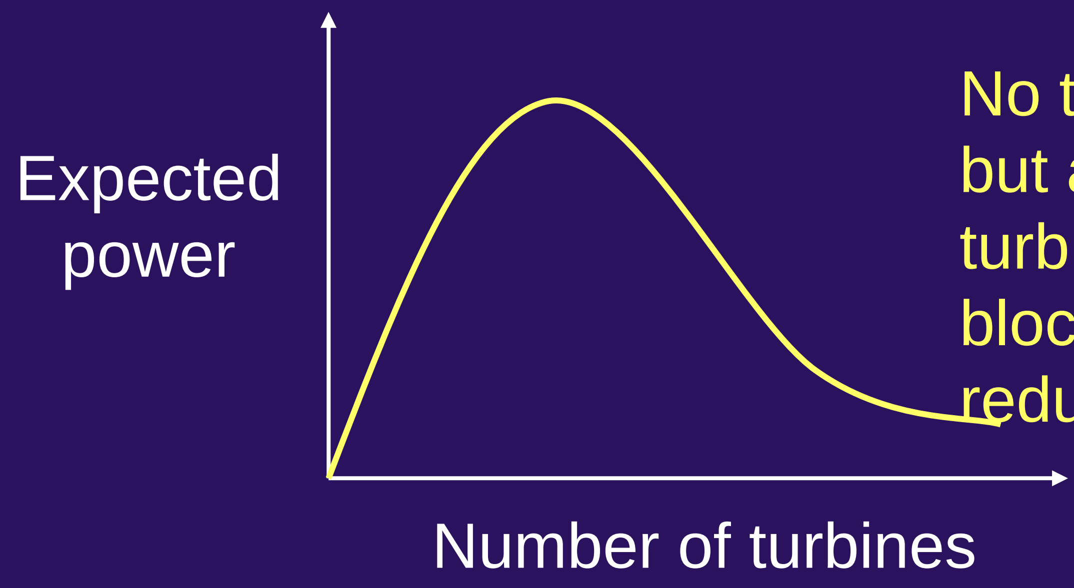
How much power is available from a tidal channel?

- The kinetic energy flux $\frac{1}{2}A\rho\overline{u^3}$ is commonly used, where *A* is the local cross-sectional area, and *u* the current.

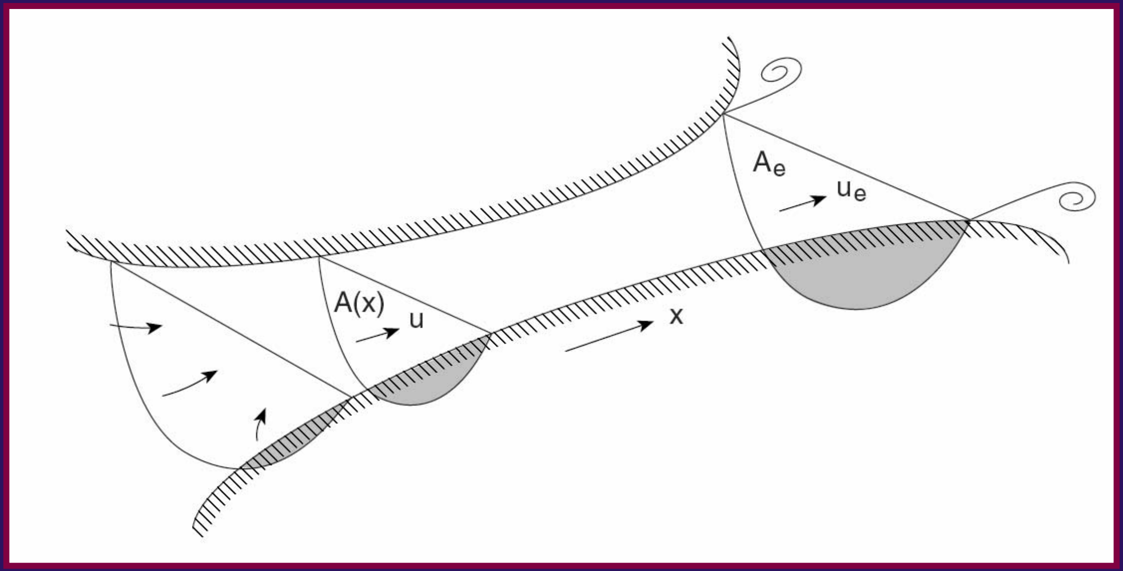
- As *Au* is fairly uniform, this estimate is very sensitive to where it is evaluated.

- There is no reason to believe that this formula provides any indication of the maximum power that can be extracted.

Maximizing power is a problem in optimization



No turbines = no power,
but adding too many turbines
blocks the flow and
reduces the power output



Model of a tidal channel

A general approach

Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \zeta}{\partial x} = -(F + F_{turb})$$

← Natural + turbine friction

Continuity, if the channel is much shorter than a wavelength:

$$A(x)u(x,t) = Q(t)$$

← Volume flux

Then we can integrate along the channel:

$$c\frac{dQ}{dt} - g\zeta_0 = -\int_0^L (F + F_{turb})\,dx - \frac{1}{2}u_e|u_e|$$

$$c = \int_0^L A^{-1}\,dx$$

Forcing

Exit speed

$$c\frac{dQ}{dt} - g\zeta_0 = -\int_0^L F_{\text{turb}}\,dx - \alpha Q|Q|$$

$$\alpha = \int_0^L C_d(hA^2)^{-1}\,dx + \frac{1}{2}A_e^{-2}$$

Solve an ordinary, not partial, differential equation

The power produced is: $$\overline{\int_0^L \rho F_{turb}\, Q\, dx} = \overline{\rho Q \int_0^L F_{turb}\, dx}$$

We want to maximize this.

An easy limit is when the natural drag dominates the acceleration.

Momentum equation: $$-g\zeta_0 = -\int_0^L F_{turb}\, dx - \alpha Q^2$$

Instantaneous power: $$\rho Q \int_0^L F_{turb}\, dx = \rho Q \left(g\zeta_0 - \alpha Q^2\right)$$

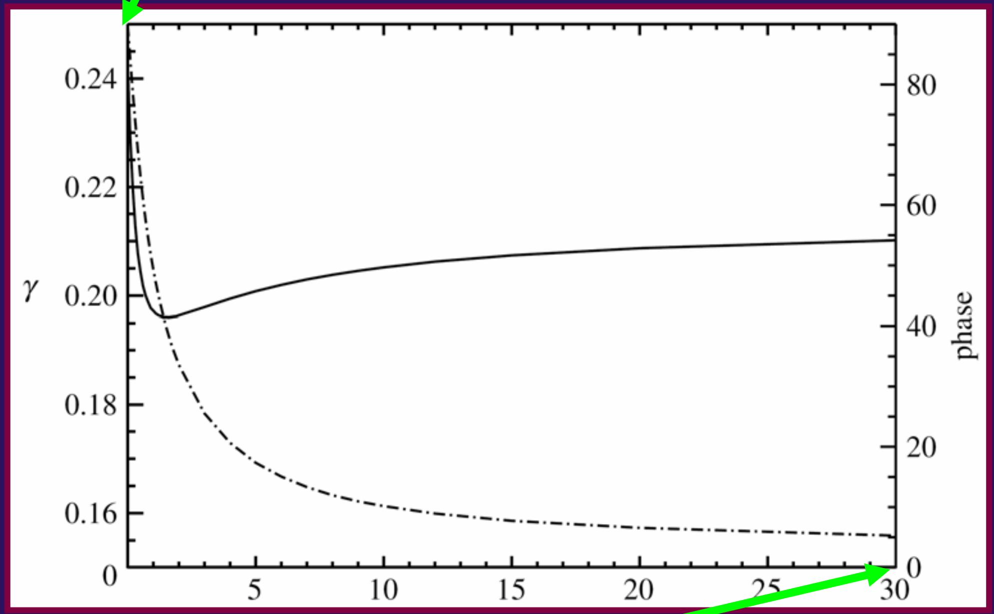
$$\zeta_0 = a\cos\omega t, \quad Q_0 = (ga/\alpha)^{1/2}$$

The maximum average power is: $$P_{max} = 0.21 \rho g a Q_0$$

This is achieved with 2/3 of the original head loss now across the turbines and Q reduced to 58% of its natural value in the absence of turbines.

In general, $$P_{max} = \gamma \rho g a Q_0$$

where *a* is the peak head, Q_0 the peak volume flux in the natural state and γ varies from 0.24 to 0.21 (with a dip to 0.196) as the regime changes from acceleration dominated, with the current lagging the head by 90^0,



to friction dominated, with a phase lag of zero.

Comments:

- The maximum power is *independent of the location of the turbines along the channel*, but assumes they are in "fences" across the whole channel with all the flow through the turbines.

- Allowance can be made for many tidal constituents.

- There may be corrections for long channels with non-uniform volume flux and for feedbacks which change the forcing.

- There is no general relation between our formula and the flux of kinetic energy, though for the special case of short channels dominated by exit separation:

$$P_{max} = 0.38 \times \frac{1}{2}\rho A_e \overline{|u_{e0}|^3}$$

Speed at exit in natural regime, probably less than in the most constricted section.

See Garrett & Cummins (2005) Proc. Roy Soc. A, **461**, 2563-2572 for more details.

Johnstone Strait and Discovery Passage

Our formula gives

1.4 GW

using

$a = 2.1$ m

$Q_0 = 310{,}000\ m^3s^{-1}$

Detailed numerical modelling by Graig Sutherland (UVic), and Mike Foreman (IOS) is being used to check assumptions and allow for splitting of the flow between Discovery Passage and Cordero Channel.

Flow diversion reduces the potential of using just one channel.

Conclusions

- A theoretical foundation for assessment of the power potential of tidal channels has been developed. Some fairly general formulae are available.
- This is being complemented by detailed numerical modelling studies to test assumptions and extend the theory.
- Further progress will be aided by collaboration between oceanographers and engineers.

The End